$p=96$, $h=91.7=92$ say, $B=94$, $B'=83$.

$\therefore t=\pi/C\,(4.3093+90.3161+409.9964)=1.76848\pi=5$ hours, 33 minutes, 21 seconds.

The actual time observed for filling this tank to within 4 inches of the top was $5\frac{1}{2}$ hours.

MISCELLANEOUS.

149. Proposed by F. P. MATZ, Ph. D., Sc. D.

Given $\sin^{-1}u+\sin^{-1}\frac{1}{2}u=\frac{1}{4}\pi$, to find u.

Solution by J. EDWARD SANDERS.

By use of the addition theorem, we have

$$\tfrac{1}{2}\sqrt{2}=u.\tfrac{1}{2}\sqrt{(4-u^2)}+\tfrac{1}{2}u.\sqrt{(1-u^2)}.$$

Squaring twice and arranging, we get the trinomial $17u^4-20u^2=-4$, or $u=\pm\sqrt{(\frac{10}{17}\pm\frac{4}{17}\sqrt{2})}$. Whence $u=\pm.50544945$......... or $\pm.95968298$..........

The first value is the one solving the question.

Also solved by R. D. Carmichael, G. W. Greenwood, A. H. Holmes, L. E. Newcomb, J. Scheffer, W. L. Tryon, G. B. M. Zerr, and the Proposer.

150. Proposed by T. N. HAUN, Mohawk, Tenn.

If $\dfrac{\sin\phi}{\sin\psi}=m$, find maximum and minimum value of $\dfrac{\sin(\phi+\theta)}{\sin(\psi+\theta)}$, where θ is known.

I. Solution by A. H. HOLMES.

$$\frac{\sin\phi}{\sin\psi}=m.\quad \therefore \sin\phi=m\sin\psi \text{ and } \cos\phi=\sqrt{(1-m^2\sin^2\psi)}.$$

$$\therefore \frac{m\sin\psi\,\cos\theta+\sin\theta\,\sqrt{(1-m^2\sin^2\psi)}}{\sin\psi\,\cos\theta+\sin\theta\,\sqrt{(1-\sin^2\psi)}}=\text{maximum or minimum.}$$

Differentiating, etc.,

$$\sin^4\psi-\frac{2\cos^2\theta(m^2+\sin^2\theta+\cos^2\theta)}{(m^2-\sin^2\theta+\cos^2\theta)^2+4\sin^2\theta\cos^2\theta}$$

$$=-\frac{\cos^4\theta}{(m^2-\sin^2\theta+\cos^2\theta)^2+4\sin^2\theta\cos^2\theta}.$$

$$\therefore \sin\psi=\frac{\cos\theta}{\sqrt{(m^2-2m\sin\theta+1)}} \text{ for maximum,}$$

$$\text{and } \sin\psi=\frac{\cos\theta}{\sqrt{(m^2+2m\sin\theta+1)}} \text{ for minimum.}$$